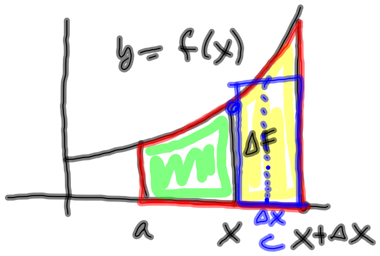
5.4b Fundamental Theorem of Calculus

proof of ftc



$$F(x) = \int_a^x f(t)\,dt$$

$$F(x+\Delta x) = \int_a^{x+\Delta x} f(t)\,dt$$

$$\lim_{\Delta x \to 0} \frac{\Delta F}{\Delta x} = \boxed{\frac{\lim_{\Delta x \to 0} F(x+\Delta x) - F(x)}{\Delta x}}$$

MVT $\boxed{f(c)\Delta x} = \Delta F$

$$\boxed{f(x) \overset{?}{=} F'(x)}$$

$$\lim_{\substack{\Delta x \to 0 \\ c \to x}} \frac{f(c)\cancel{\Delta x}}{\cancel{\Delta x}} = f(x)$$

$$\boxed{f(x) = \frac{d}{dx}\int_a^x f(t)\,dt}$$

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1.1 1.2 1.3 *Unsaved

$$\frac{d}{dx}\left(\int_{-\pi}^{x} (\cos(t))\,dt\right) = \quad \cos x$$

$$\frac{d}{dx}\left(\int_{0}^{x} \left(\frac{1}{1+t^2}\right)dt\right) = \quad \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\tan^{-1} t \Big|_0^x\right)$$

$$\frac{d}{dx}\left(\tan^{-1} x - \tan^{-1} 0\right)$$

$$\frac{1}{1+x^2} - 0$$

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1.2 1.3 1.4 *Unsaved

$$\frac{d}{dx}\left(\int_1^{x^2}(\cos(t))\,dt\right) = \cos(x^2)\cdot 2x$$

$$\frac{d}{dx}\int_2^{x^2+1}\tan^{-1}t\,dt = 2x\cdot\tan^{-1}(x^2+1)$$

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1.3 1.4 1.5 *Unsaved

Find dy/dx

$$y=\int_x^5(3t\sin(t))\,dt = -\int_5^x 3t\sin t\,dt \qquad \frac{dy}{dx} = -3x\sin x$$

$$y=\int_{2x}^{x^2}\left(\frac{1}{2+e^t}\right)dt = \int_{2x}^{a}\frac{1}{2+e^t}\,dt + \int_a^{x^2}\frac{1}{2+e^t}\,dt$$

$$\frac{dy}{dx} = -\frac{2}{2+e^{2x}} + \frac{2x}{2+e^{x^2}}$$

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Find a function $y = f(x)$ with derivative $dy/dx = \tan(x)$ that satisfies the condition $f(3)=5$. Graph the function.

$$y = \int_a^x \tan(t)\,dt$$

$$\frac{dy}{dx} = \tan x + 0$$

$$y = \int_3^x \tan(t)\,dt + 5$$

$$f(3) = 0 + 5$$

$$\downarrow$$

$$\int_3^3$$

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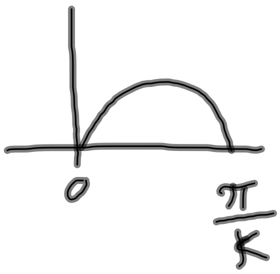
63.

$y = \sin kx$ period $= \frac{2\pi}{k}$



$$\int_0^{\pi/k} \sin(kx)\,dx = \frac{2}{k}$$

$$-\frac{1}{k}\cos(kx)\Big|_0^{\pi/k}$$

$$-\frac{1}{k}(\cos\pi - \cos 0)$$

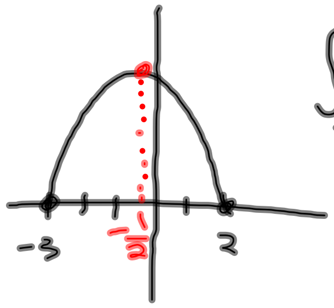
$$-\frac{1}{k}(-1-1) = \frac{2}{k}$$

$$\frac{1}{\cancel{k}}\cdot\sin(kx)\cdot\cancel{k}$$

Nov 17-9:17 AM

64.

$y = 6 - x - x^2 \qquad -3 \le x \le 2$



$\int_{-3}^{2} 6 - x - x^2 \, dx = \frac{125}{6}$

$6x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-3}^{2}$

$\left(6 \cdot 2 - \frac{2^2}{2} - \frac{2^3}{3}\right) - \left(6(-3) - \frac{(-3)^2}{2} - \frac{(-3)^3}{3}\right)$

$A = \frac{2}{3} b \cdot h$

$b = 5$

$h = 6 - \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)^2$

$h = \frac{25}{4} = 6.25$

$A = \frac{2}{3} \cdot 5 \cdot \frac{25}{4} = \frac{125}{6}$

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